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A THEORETICAL ANALYSIS OF THE DEMAND FOR EMISSION LICENSES

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ABSTRACT

The issue of how firms with inputs of variable quality will react in a market for transferable emissions licenses is analyzed. First, it is shown that the derived demand for licenses will, in general, be downward sloping. This is followed by a discussion of the effects of imperfections in product and factor markets on abatement decisions.

This paper examines the qualitative effects that a market in transferable licenses in emissions will have on a firm's input decisions and its expenditure on abatement equipment. The case of the competitive firm is examined in detail, and this is compared with a firm which can exert monopoly power in product and factor markets. The model employed here differs from previous work in that the price of the variable input is explicitly related to its quality. This can be compared with the more conventional approach which treats the pollutant as a factor of production.¹ Several authors have shown that the derived demand for inputs of fixed price and quality are downward sloping.² In Section 1, this result is extended to the case where input quality can be varied. Section 2 compares the demand for licenses under competition with the demand for licenses when a firm can exert power over product or factor markets. In Section 3, the role of other traders and the authority issuing licenses is explicitly included in the analysis. Section 4 summarizes the results.

1. The General Problem

Attention is focused on the problem of controlling emissions associated with the use of productive inputs. When the relationship between emissions and ambient pollutant concentrations is linear, then the subsequent analysis obtains for the control of secondary pollutants as well as the control of primary emissions.

The control of sulfur oxides emissions is one example for which the model would be appropriate. Sulfur enters into the production process through the use of natural resources that contain it, usually coal and petroleum used as energy inputs. When these inputs are burned some of the sulfur contained in them is converted to SO_2 and SO_3 . For a given abatement technology, the relationship between sulfur entering the production process and resulting emissions of sulfur oxides is approximately linear.

The firm may adopt two basic approaches to reducing emissions. It can either reduce emissions directly by purchasing equipment such as scrubbers and baghouses or it can reduce the level of pollutant entering into the production process. This latter reduction is normally accomplished by purchasing higher quality inputs, which typically cost more, by curtailing output, or by varying the amount of inputs used per unit of output in production. For simplicity, the last method for reducing emissions will be ignored. Suppose that the firm has a production function $f(E)$, where E represents the level of inputs. The function f is assumed to be twice differentiable and strictly concave so that $f' > 0$ and $f'' < 0$.

Let $X(R,s,E)$ characterize the firm's abatement opportunities. X is the total annual emission rate; R is the total annual expenditure on abatement; and s is the amount of the pollutant contained in a unit of the input stream, E . Emissions are assumed to decrease with greater abatement expenditures, but there are decreasing returns to

such endeavors, (i.e., $X_1 < 0$ and $X_{11} > 0$). On the other hand, annual emissions will increase if the firm chooses lower quality inputs or increases the level of its inputs (i.e., $X_2 > 0$ and $X_3 > 0$). Furthermore, it will be assumed that increasing inputs will not improve the marginal effect of a given pollutant content, and may make it worse (i.e., $X_{23} \geq 0$).³ The firm's problem is to maximize profits, or the difference between total revenues and the sum of input costs, abatement costs and license costs. Formally, we have:

$$\text{Maximize}_{R,s,E} pf(E) - e(s)E - wX(R,s,E) - R \quad (1)$$

where

p = price of output,

$e(s)$ = unit price of inputs; $e' < 0$ $e'' > 0$, and

w = license price.

The price of inputs is presumed to be a convex function of the pollutant content. From this, it immediately follows that a firm would never wish to use two or more different quality inputs simultaneously, where such inputs are defined solely in terms of pollutant content.⁴ Empirically, this relationship has been shown to hold approximately for heavy fuel oil prices in Los Angeles.⁵

First-order conditions for an interior solution are given by:

$$-wX_1 - 1 = 0 \quad (2)$$

$$-e'E - wX_2 = 0 \quad (3)$$

$$pf' - e - wX_3 = 0 \quad (4)$$

Equation (2) says that at the margin, an additional dollar spent on abatement equipment will be exactly offset by the savings resulting from decreased emissions. Equation (3) balances the reduction in emissions from buying higher quality inputs against the increase in the cost of buying licenses. Equation (4) equates the marginal revenue product of using an additional unit of inputs with the increase in the cost of input, which consists of two components: the direct cost of inputs, e , and the indirect cost due to having to purchase more licenses, wX_3 .

The interesting comparative statics questions revolve around the effect of a change in the license price on abatement expenditures, the pollutant content of inputs, the level of inputs, and hence, the ultimate level of emissions which is chosen. Totally differentiating the first order conditions gives rise to the following Hessian matrix, C :

$$C = \begin{bmatrix} -wX_{11} & -wX_{12} & -wX_{13} \\ -wX_{12} & (-e''E - wX_{22}) & (-e' - wX_{23}) \\ -wX_{13} & (-e' - wX_{23}) & (pf'' - wX_{33}) \end{bmatrix}$$

Let C_{ij} denote the ij th cofactor of C and $[C]$ denote the determinant. Performing the comparative statics yields expressions for the effect of a change in license price on the endogenous variables:

$$\frac{\partial R}{\partial w} = \frac{1}{[C]} [C_{11}X_1 + C_{12}X_2 + C_{13}X_3] \quad (5)$$

$$\frac{\partial s}{\partial w} = \frac{1}{[C]} [C_{12}X_1 + C_{22}X_2 + C_{23}X_3] \quad (6)$$

$$\frac{\partial E}{\partial w} = \frac{1}{[C]} [C_{13}X_1 + C_{23}X_2 + C_{33}X_3] \quad (7)$$

$$\frac{\partial X}{\partial w} = X_1 \frac{\partial R}{\partial w} + X_2 \frac{\partial s}{\partial w} + X_3 \frac{\partial E}{\partial w} \quad (8)$$

Assume that sufficiency conditions for an interior maximum are met.⁶ This implies that C is negative definite. Even with this assumption, $\frac{\partial R}{\partial w}$, $\frac{\partial s}{\partial w}$ and $\frac{\partial E}{\partial w}$ cannot be signed unambiguously. However, it is possible to show that the demand for licenses is downward sloping (i.e., $\frac{\partial X}{\partial w} < 0$). Substituting equations (5) - (7) into (8) yields:

$$\frac{\partial X}{\partial w} = \frac{1}{[C]} \begin{bmatrix} (X_1, X_2, X_3) \\ \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \\ \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \end{bmatrix} \quad (9)$$

Because C is negative definite, this implies C^{-1} is negative definite. Thus, equation (9) indicates that $\frac{\partial X}{\partial w} < 0$.

While the sign of the terms in equations (5) - (7) cannot be determined exactly, it is possible to infer from equation (9) that an

increase in the price of a license will induce at least one of the following events: (1) an increase in the level of annual abatement expenditures, (2) a decrease in the pollutant content of inputs or (3) a decrease in the level of inputs. Of course, it is possible that more than one of these events will occur in response to a license price increase, but at least one such event must occur.

The result derived here concerning the downward sloping demand curves also holds for the case in which the level of inputs are fixed, but the quality is allowed to vary. This latter case may be applicable to several firms in the short run. A case in point would be electric utilities who burn high sulfur residual fuel oils. The only difference between the case when inputs are constrained and the more general case is that in the constrained case, an increase in the license price will lead to an increase in abatement expenditures or a decrease in the pollutant content of inputs, and possibly both.

It is a straightforward matter to show a monopolist will have a downward sloping derived demand for licenses in this general case. However, at this level of analysis, it is not obvious how the demand by a competitive firm compares with the demand by a firm that can exert market power. To allow for a case by case comparison, it is helpful to consider a less general formulation. This is the subject of the next section.

2. A Comparison of Competition with Market Power

A simple case to analyze is where the pollutant in the inputs just equals emissions; that is, no abatement can be achieved through expenditure on equipment. In this case, reductions can be achieved by reducing the pollutant content of inputs and/or reducing the level of inputs. One example would be the containment of sulfur oxides through the purchase of lower sulfur fuels. Formally, the firm's problem may be written as follows:

$$\text{Maximize } pf(E) - e(s)E - wsE \quad (10)$$

$$s, E$$

First-order conditions for an interior maximum are given by:

$$-e'E - wE = 0 \quad (11)$$

$$pf' - e - ws = 0. \quad (12)$$

Equation (11) indicates that s should be chosen so as to equate the cost of polluting more, w , with the marginal cost of buying higher quality inputs, $-e'(s)$. Equation (12) balances the marginal revenue product with an increase in input costs.

Define B to be the Hessian associated with (10). Then,

$$B = \begin{bmatrix} -e''(s)E & 0 \\ 0 & pf'' \end{bmatrix} \quad (13)$$

From the assumptions on e and f , B is negative definite. An

examination of the effects of a change in the price of a license on pollutant content and the overall level of inputs yields:

$$\frac{\partial s}{\partial w} = -\frac{1}{e''(s)} < 0 \quad (14)$$

$$\frac{\partial E}{\partial w} = \frac{s}{pf''} < 0 \quad (15)$$

Equation (14) says that the pollutant content decreases with an increase in the price of a license while (15) says that the level of inputs also declines. Since the overall level of emissions is given by sE , it is readily seen that emissions decrease in response to an increase in the price of a license.

It is possible to compare the situation when the firm can exert market power with the competitive case by making suitable changes in (10) and carrying out the required optimization. Three cases will be considered: first, the case of pure monopoly; next, the case when a firm exerts some influence over the energy market and finally, the case when a firm can dominate the license market. The monopolist's problem is the same as above, except now $p = p(f(E))$, which gives:

$$\text{Maximize}_{s,E} p(f(E))f(E) - e(s)E - wsE \quad (16)$$

First-order conditions for an interior maximum are given by:

$$-e'(s)E - wE = 0 \quad (17)$$

$$pf' + fp'f' - e(s) - ws = 0 \quad (18)$$

Equations (17) is identical with equation (11). From the assumptions on e , the value for s which solves (17) (assuming one exists) will be unique.⁷ Thus, the monopolist and perfect competitor will choose the same pollutant content. To determine who would pollute more, it is only necessary to consider whether the monopolist will use more or fewer inputs than in the competitive case. Assuming the revenue function for the monopolist is strictly concave and an interior solution to the problem exists, then the monopolist will use less energy and, hence, pollute less than his competitive counterpart. To see this, define the revenue function: $R(E) = p(f(E))f(E)$. The usual differentiability assumptions imply $R' > 0$ and $R'' < 0$. Comparing conditions (12) and (18), it is clear that setting E at the optimal level in the competitive case will yield the following inequality:

$$pf' + fp'f' < e(s) + ws, \quad (19)$$

since $fp'f' < 0$. The question is whether (19) can be brought into equality by adjusting E . From (11) and (17), we saw that the pollutant content is identical for the two cases, independent of the level of inputs which is chosen. This means that the expression on the right-hand side of (19) can be treated as a constant. Noting that the left-hand side of (19) equals $R'(E)$, it immediately follows that the only way to bring (19) back into equality is to decrease E from the competitive level.

So far, we have derived conditions under which the monopolist will emit less and produce less than in the perfectly competitive case. The key assumption concerned the shape of the revenue function. This assumption is also critical for deriving the comparative statics results given below:

$$\frac{\partial s}{\partial w} = -\frac{1}{e''(s)} < 0 \quad (20)$$

$$\frac{\partial E}{\partial w} = \frac{s}{R''(E)} < 0 \quad (21)$$

A comparison of Equations (14) and (20) reveals that the effect of a change in license price on pollutant content will be the same for the monopolist and the competitive firm for a given level of input quality. The effect of a change in license price on input usage will, in general, differ, even for inputs of the same quality. However, the analysis reveals that the qualitative results under monopoly and competition are the same. Both pollutant content and input usage decline with an increase in the price of a license.

The results for the case in which the firm faces an upward sloping supply curve for inputs closely parallel the monopoly case. The problem is the same as the competitive case except e is now a function of s and E . The firm tries to:

$$\text{Maximize } pf(E) - e(s,E)E - wsE. \quad (22)$$

s, E

The price of inputs is assumed to increase as demand increases

($e_2 > 0$). In addition, it will be assumed that changing the pollutant content will have no influence on the relationship between input demand and price ($e_{12} = 0$). This latter assumption essentially allows the solution to the first-order conditions to proceed in two stages. First, the pollutant content is determined, and then the level of inputs is chosen.

First order conditions for an interior maximum to (22) are given by:

$$-e_1E - wE = 0 \quad (23)$$

$$pf' - e - Ee_2 - ws = 0 \quad (24)$$

Equation (23) determines the optimal pollutant content, s . If E is set to the optimal competitive level, this gives rise to the following inequality:

$$pf' - Ee_2 < e + ws \quad (25)$$

The problem is to adjust E so as to bring (24) into equality so that the first order conditions are satisfied. Assuming that the costs of inputs eE , is a convex function in E (for any given s) is sufficient to insure that the optimal level of inputs will be less than the competitive case.

The problem of assessing the behavior of a firm which can exert control over the market price for emissions licenses is similar to the previous case, but somewhat more complex. The general problem

is the same as in the competitive case except now license price is presumed to be negatively related to emissions so that $w = w(sE)$ and $w' > 0$. The conventional approach to such problems is to disregard output effects and solve the following cost minimization.

$$\underset{s}{\text{Minimize}} C(s) = e(s)\bar{E} + w(s\bar{E})s\bar{E}, \quad (26)$$

where the level of inputs is fixed at \bar{E} . There are two basic reasons for ignoring output effects: first, because the comparative statics results are ambiguous when these effects are included, and secondly, because output effects may not be very important in the short-run.

Dividing (26) by \bar{E} and solving the equivalent minimization problem yields the following first order condition:

$$e'(s) + w + s\bar{E}w' = 0 \quad (27)$$

Equation (27) balances the marginal cost of buying more licenses, $w + s\bar{E}w'$, with the cost of buying lower sulfur fuel. If the cost function, $C(s)$, is convex so that $C''(s) \geq 0$, then the optimal pollutant content chosen will be less than in the competitive case, provided the output produced is the same. The argument parallels the case of monopoly and will not be repeated here. Instead, we turn to an alternative formulation of the market power problem which explicitly considers the role of other agents.

3. Market Power: A More General Approach

The subsequent analysis considers the case where one agent exercises market power, while all other agents assume they cannot affect the price of a license or the quantity of licenses issued, L , (i.e., a Stackelberg "leader and follower" model). The aggregate reported demand curve for all agents excluding i is denoted by $Q^{-i}(w)$; it is assumed that Q^{-i} is twice continuously differentiable and downward sloping, i.e., $Q^{-i'} < 0$. Let $Q(w)$ represent the aggregation of i 's true demand for licenses, $Q_i(w)$, with $Q^{-i}(w)$, which i takes as given. The quantity of licenses supplied by the "center" is given by $C(w)$ which is presumed to be twice continuously differentiable and strictly increasing, i.e., $C' > 0$. The curves are illustrated in Figure 1.

Agent i is aware that he may choose any point on the center's supply curve above the price of w_0 , which represents the equilibrium price if i submits no demand. A price of w_1 , assumed to be greater than w_0 , would result if i submitted his true demand.

To derive i 's best approach to the problem, first note that his effective supply, denoted as $S(w)$ is given by:

$$S(w) = C(w) - Q^{-i}(w) \quad \text{for } w \geq w_0 \quad (28)$$

Because $C' > 0$ and $Q^{-i'} < 0$, $S'(w) > 0$, which means that agent i 's effective supply curve of licenses to i is strictly increasing.

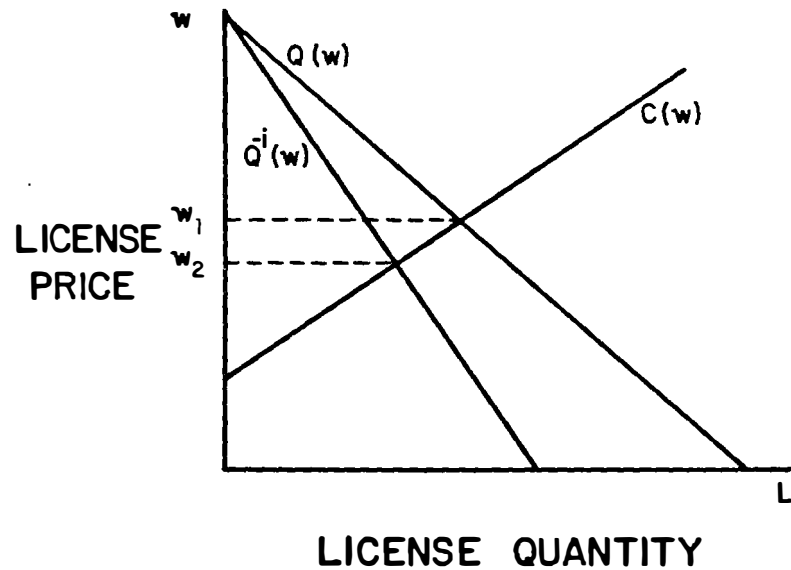


FIGURE 1

The General Supply and Demand Problem

Define the inverse of $S(w)$ as $s(L)$. Since S is upward sloping, so is its inverse, i.e.,

$$w = s(L) \quad s' > 0 \quad (29)$$

Finally, define agent i 's inverse demand function as $d_i(L)$; this function is presumed to be strictly decreasing, i.e., $d_i' < 0$. Agent i 's problem is depicted in Figure 2. L_1 represents the quantity of licenses agent i receives if he reveals his true demand and the market clears at w_1 .

The question which i must address is whether it is in his interest to misstate his true demand, and if so, in which direction. To answer this question i 's interest is defined as follows:

$$\text{Agent } i\text{'s net gain} = \int_0^L d_i(q) dq - s(L)L \quad (30)$$

Equation (30) says that the gain i derives by purchasing L licenses is given by the difference between the area under his inverse demand curve between 0 and L and the costs of purchasing L licenses. With this measure of welfare, it is apparent that agent i will never demand more than L_1 licenses since he not only has to pay more for all inframarginal units, but he also loses on the marginal units as well. The only other possibility is that agent i demands fewer than L_1 licenses. Suppose that he chooses a level of licenses equal to L_2 as illustrated in Figure 2. To compare this outcome to the situation in which i receives L_1 licenses, it is convenient to sort out his gains

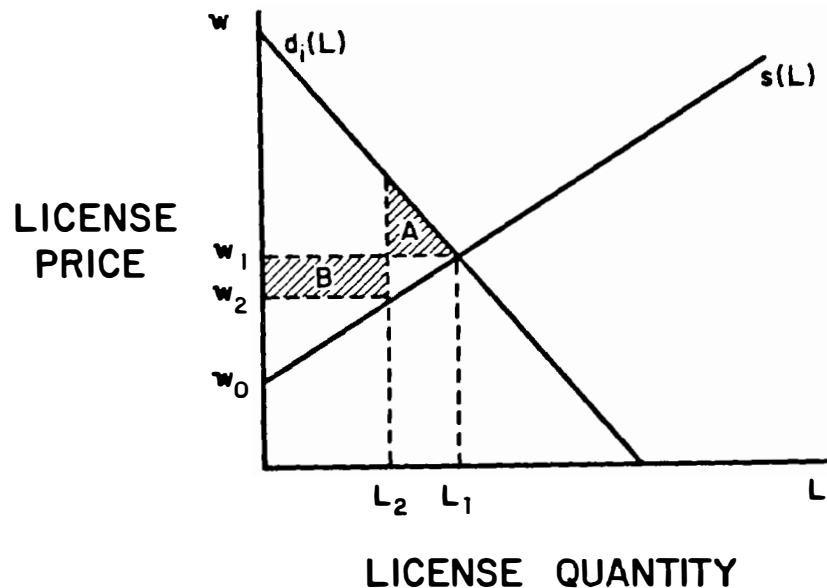


FIGURE 2
Agent i's Problem

and losses in a systematic manner. The gains to i which result from being charged a price w_2 instead of w_1 are noted by the shaded area B. His losses due to the fact he purchases $(L_1 - L_2)$ fewer licenses are represented by area A. If $(B - A)$ is positive, then we may conclude that i 's welfare associated with (L_2, w_2) exceeds that associated with revealing his truthful demand, (L_1, w_1) . The problem of showing that it is always in i 's interest to overabate is equivalent to showing that there exists an $L \in (0, L_1)$ for which $(B - A)$ is positive.

Maximizing (30) with respect to L and assuming an interior maximum exists yields the following first order condition:

$$d_i(L)(s(L) + Ls'(L)) = 0 \quad (31)$$

Noting $s'(L) > 0$ implies:

$$d_i(L_1) < s(L_1) + L_1 s'(L_1) \quad (32)$$

To bring (32) back into equality requires that the L selected be less than L_1 . This shows that it is in agent i 's interest to underrepresent his demand for pollution emission provided that there is no subsequent trading of licenses, agent i knows the demand curve of all other agents and the supply curve of the center, and the second order conditions are satisfied. It is of some importance to know what conditions on the demand or supply curve would guarantee that the stationary point is a local maximum. The second order sufficiency conditions require:

$$d'_i(L) - 2s'(L) - Ls''(L) < 0 \quad (33)$$

From (33), we see that it is sufficient to presume that the rate of change of the slope of the effective supply curve, $s''(L)$, is nonnegative.⁸

The problem analyzed above parallels the case of pure monoposony very closely. The only difference is that agent i is not the only buyer, and hence, must consider how the demand of others will affect his supply. The qualitative results which emerge in the two problems are the same, namely that output and price are both below the level they would have reached in the presence of competition.

The extreme cases were not considered in the analysis. If agent i 's effective supply curve does not vary with price, then he will demand L_1 licenses since, by assumption, he cannot exert any downward pressure on the price of a license. In this case i would perceive the license market in the same light as an emissions tax. Another case not considered is when the center fixes the supply of licenses so that $C'(w) = 0$. In this case, the result still obtains that the firms with market power will overabate.

The principal result is called into question, however, when any "real world" considerations are brought to bear on the problem. For example, an incomplete knowledge of others' demand curves and the center's supply curve would mean that agent i would have to guess at the equilibrium price in his absence. Of course, knowing the

equilibrium price is not enough. Agent i cannot construct his effective supply curve without knowing the center's supply and others' demands over a fairly wide range. The addition of secondary markets further complicates the issue. The clearing price expected in the secondary markets is likely to vary across agents and will affect each individual's behavior in the initial auction. Without explicit modeling of such problems, it is a little premature to conclude that market power will result in overabatement.

4. Conclusions

The analysis focused on the derived demand for tradable licenses. In the general case it was found that introducing inputs of different quality did not change the basic result that the derived demand was downward sloping. This holds both for the monopolist and the competitive firm. A comparison of three cases of market power in a more restricted setting revealed that in all three cases, firms would tend to overabate in comparison to the competitive firm. A more general analysis of the case when a firm can dominate the license market indicated that the assumptions required to obtain the overabatement result may be too restrictive. This is one area which merits further thought if marketable permits are to become a reality.

Footnotes

*The work reported here was supported in part by the California Air Resources Board. I wish to thank Roger Noll and James Quirk for providing helpful comments. All views and conclusions expressed herein are my responsibility.

1. For example, see Baumol and Oates (1975), p. 35ff.
2. For examples, see Samuelson (1974), pp. 76-78, Russell (1964) and Winch (1965).
3. This assumption can be explained in terms of the desulfurization of fuel oil. Suppose the effect of desulfurization is to remove a constant fraction $(1 - \frac{1}{n})$ of total potential emissions, sE . Total expenditure on abatement is constant by assumption. The problem is to consider how $\frac{\partial X}{\partial s}$ changes as inputs increase. Consider a discrete change in inputs from E to $(E + \Delta E)$. Before the change, $\frac{\Delta X}{\Delta s} = \frac{1}{n}\Delta s E$. After the change $\frac{\Delta X}{\Delta s} = \frac{1}{n}\Delta s (E + \Delta E)$. In the limit, it is apparent that $X_{23} \geq 0$.
4. The proof is straightforward. Suppose the firm wishes to use two different inputs with respective costs $e(s_1)$ and $e(s_2)$. Let λ equal the fraction spent on the first type and $(1-\lambda)$ be the fraction spent on the second. Then, the average cost of inputs would be $[\lambda e(s_1) + (1-\lambda)e(s_2)] > e(\lambda s_1 + (1-\lambda)s_2)$. Thus, using inputs of the same quality with the equivalent pollutant content would be cheaper.

If the firm wishes to purchase n different quality inputs, where n is arbitrary, the same line of reasoning holds.

The proof assumes, of course, that any convex combination of pollutant contents are available for values of λ on the unit interval. In the case of sulfur in fuel oil, this is a reasonable approximation.

5. On this point, see Chapter 3 of "Implementing Tradable Emission Licenses: Sulfur Oxides in the Los Angeles Air Shed," written by William Rogerson.

6. For the problem to make sense, R , s , and E must be nonnegative. These constraints are assumed to be ineffective.

7. For example, if $\lim_{s \rightarrow 0} e'(s) = +\infty$ and $\lim_{s \rightarrow +\infty} e'(s) = 0$ (i.e., e is a "neoclassical" function), then for any $w > 0$, (17) has a unique positive solution in s .

8. In the economics literature the abatement cost function for all firms is typically presumed to be twice differentiable and strictly convex. Accepting this assumption would mean that a sufficient condition for a global maximum on $(0, L_1)$ would be that $C''(w) \geq 0$. For a specific example, see Ackerman, p. 279.

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